Business Logic: A Simple Mathematical Approach

Any manufacturing unit, irrespective of the product, starts operation with a deficit fund management. Any start-up business's primary struggle is about reaching the break-even first, and then erasing the deficit gradually. The final objective is of course achieving full independence from all debts, and continuing operations without any extra capital from outside being pumped in.

Apart from this, all such businesses have a certain amount of internal consumption of its own products. For example, the workers of a food production unit never buy their own food, or workers of a bicycle company are usually riding bicycles to work and back home that the company produces in its own factory. These internally consumed units have no market value as such. With these basic assumptions we present the following linear equation.

Let us now define the following variables.

Overhead of a business per production cycle = X {In reality this X is a function of the number of units in N that cycle produces, but the X doesn't change with every ΔN change, but takes a quantum jump after N reaches the top capacity of that unit, denoted by $\dot{N}$, say.)

Every unit takes inventory of cost D.

The internal consumption is the threshold T. Now N≥T, otherwise there is no point in doing business at all.

Let's assume N=T+ν, so that ν (nu) is the number of sellable units, sold at the market rate m/unit. Therefore,

C= Cost per cycle = D(T+ν)+X; and Revenue per cycle is m.ν

Profit **P=m.ν – D(T+ν) – X** → Eqn. 1

Now, $0 \leq \nu \leq \dot{N} - T$ makes ν a bounded variable with respect to any given X.

It's easy to find out the 'break-even' point here, where P=0. This means **ν=(X+DT)/(m-D)**. In our test case, a start-up food delivery unit also donating food to a few homeless and hapless kids, D=27.53, T=28, m=60, and X=560.43; and this returns ν=41, implies that 41 units have to be sold profitably @60.00 in order to feed the 28 kids for free. Let's denote this number as ν_0.

Now, that was cakewalk. But the real challenge is that the business must have taken some time to reach the break-even, a few productions cycles have gone by, and negative profit has piled up. This negative profit made, loss in simple terms, is the real deterrent for the business to grow any further. From the initial capital introduced or from the capital periodically pumped in the production cost is covered, but what if the piled up debt/loss surpasses the profit made so far? Simple, production will stop, because there will be no inventory. Hence, we introduce the concept of

1. **CP_n = Cumulative Profit after n cycles = $_0\Sigma^n P_n$**
2. **F = Freedom (or production readiness) = CP_n-C_{n+1},** where C_{n+1} is the estimated cost at the (n+1)-th cycle.

Is the business ready for going to the next production cycle, irrespective of any extra capital pump-in, or not? If **$F \geq 0$**, we can safely assume that the business is healthy, otherwise it's not.

Now this is a tricky situation. The variable ν is a random variable. Despite aggressive marketing and other strategies adopted, one can never make sure that a fixed value of ν will be attained on any given day. However, the variable is having a finite median, being a bounded random variable, and is therefore integrable. Let's now go back to the equation.

$F = {}_0\Sigma^n P_n = {}_0\Sigma^n(m.\nu - D(T+\nu) - X) = (m-D)\,{}_0\Sigma^n \nu_n - n(DT+X)$

Hence, $F \geq 0 \rightarrow \mathbf{{}_0\Sigma^n \nu_n \geq n\nu_0}$ → Eqn. 2

Here, $\mathbf{{}_0\Sigma^n \nu_n}$ is the partial sum of a random variable ν up to the n-th trial. There is no one definite method of calculating this sum, as it can be treated as a Bernoulli distribution, or as a Poisson Binomial, or even as a Discrete Fourier Transform. Several attempts have been made in last 5-6 years to solve this deceptively simple looking problem, but all the results are case-specific, i.e. no general prescription is there yet.

However, to simplify, we resort to the Wald's identity which deals in an estimate of the sum and says that

$\mathbf{E[{}_0\Sigma^n \nu_n] = E[n].E[\nu_1]}$ → Eqn. 3

Therefore, $F \geq 0 \rightarrow \mathbf{E[n].E[\nu_1] = \{n(n+1)/2n\}.E[\nu_1] \geq n\nu_0}$

Or, $\mathbf{E[\nu_1] \geq \{2n/(n+1)\}.\nu_0}$ → Eqn. 4

However, this is true for sufficiently large n. F=0 can be attained much earlier than this, but it is also seen that stopping capitalizing at that very point may not be wise, because the cumulative profit at that point may not be large enough to cover all old debts AND the upcoming production/inventory expenses, and hence it may bounce back to the 'Sagging third' in no time. In our test case, we have observed that the F =0 can be reached in the 19^{th} week, while the actual independence comes after the 25^{th} week of operations. If getting credit from the suppliers becomes uncertain for some reason, this lag will be reason enough for nosedive. It's better to go by Eqn. 4 therefore.